

Cosmological constraints on the Hu-Sawicki modified gravity scenario

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In this paper we place new constraints on a $f(R)$ modified gravity model recently proposed by Hu and Sawicki. After checking that the Hu and Sawicki model produces a viable cosmology, i.e. a matter dominated epoch followed by a late-time acceleration, we constrain some of its parameters by using recent observations from the UNION compilation of luminosity distances of Supernovae type Ia, including complementary information from Baryonic Acoustic Oscillations, Hubble expansion, and age data. We found that the data considered is unable to place significant constraints on the model parameters and we discuss the impact of a different assumption of the background model in cosmic parameters inference.

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I. INTRODUCTION

The recent cosmological data from Cosmic Microwave Background Anisotropies, galaxy surveys and luminosity distance of type Ia supernovae are all providing supporting evidence for a dark energy component, responsible for more than 70% of the total energy budget in our universe (see e.g. [1]). Several candidates have been suggested for explaining this component, as, for example, minimally coupled scalar fields (see e.g. [2] and references therein). However, it may also be that the cosmological evidence for acceleration comes from the wrong assumption of general relativity, i.e. that no dark component is present but actually a modification to gravity is at work. In this respect, $f(R)$ theories seem to provide a quite large number of viable models (for a recent review see [3] and [4]). A particular $f(R)$ model that evades solar system test has been proposed by Hu and Sawicki ([5], HS hereafter). The model has a modified Einstein-Hilbert action:

$$S = \int d^4x \sqrt{-g} \left[\frac{R + f(R)}{2k^2} + L_m \right] \quad (1)$$

L_m is the matter lagrangian, $k^2 = 8\pi G$ and

$$f(R) = -m^2 \frac{c_1 \left(\frac{R}{m^2} \right)^n}{1 + c_2 \left(\frac{R}{m^2} \right)^n} \quad (2)$$

with $m^2 = k^2\rho/3$ and c_1 , c_2 and n as free parameters.

As shown in [5] this model is able to reproduce the late time accelerated universe but with distinctive deviations from a cosmological constant. In this paper we investigate the cosmological viability of the HS model in more detail. After a brief description of the model, in the next section we will show that the HS model satisfies indeed the general conditions presented by [6] as a viable $f(R)$ model. In Sec. III we compare the HS model with current data from SN-Ia luminosity distances from the UNION catalog ([8]), Baryonic Acoustic Oscillation data ([11]) and age constraints from the analysis of Simon, Verde and Jimenez ([9]). We show that the current data is fully compatible with the HS model and that, unless a prior on the matter density is used, the parameters of the model are unconstrained. In particular, we analyze the impact of the HS model in the determination of the current matter density. As we will show, assuming the HS model instead of the standard cosmological model, could relax the constraints on the effective matter density.

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A future incompatibility between the values of the matter density Ω_M determined from different datasets and under the assumption of the standard Λ CDM model could therefore provide an hint for a modified gravity scenario.

II. THE HU-SAWICKY MODEL

Let us briefly review in this section the basic equations and results of the HS model. Varying the action in Eq. 1 with respect to the metric $g^{\mu\nu}$ one obtains the modified Einstein equations:

$$G_{\mu\nu} + f_R R_{\mu\nu} - \left(\frac{f}{2} - \square f_R\right) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R = k^2 T_{\mu\nu} \quad (3)$$

where $f_R = df/dR$ and $f_{RR} = d^2 f/dR^2$ and assuming a flat FRW metric, the modified Friedmann equation:

$$H^2 - f_R(HH' + H^2) + \frac{f}{6} + H^2 f_{RR} R' = \frac{k^2 \rho}{3} \quad (4)$$

with $' \equiv d/d\ln a$ and ρ the matter density at present time.

Defining the new variables $y_H = (H^2/m^2) - a^{-3}$ and $y_R = (R/m^2) - 3a^{-3}$ the Friedmann equations can be expanded in a system of two ordinary differential equations:

$$y'_H = \frac{y_R}{3} - 4y_H \quad (5)$$

$$y'_R = 9a^{-3} - \frac{1}{y_H + a^{-3}} \frac{1}{m^2 f_{RR}} \left[y_H - f_R \left(\frac{y_R}{6} - y_H - \frac{a^{-3}}{2} \right) + \frac{f}{6m^2} \right] \quad (6)$$

In order to compare the HS model with the cosmological constraints usually derived under the assumption of dark energy, it is useful to introduce an *effective* dark energy component with present energy density $\tilde{\Omega}_x = 1 - \tilde{\Omega}_m$ and equation of state w_x , where $\tilde{\Omega}_m$ is the effective matter energy density at present time.

Of course, in reality no dark energy component is present, the only component present is matter and modified gravity gives the acceleration. Considering the Friedmann equation:

$$\frac{H^2}{H_0^2} = \frac{\tilde{\Omega}_m}{a^3} + \tilde{\Omega}_x e^{\int_a^1 da \frac{3[1+w_x(a)]}{a}} \quad (7)$$

the effective equation of state parameter w_x for the dark energy component is given by

$$w_x = -1 - \frac{1}{3} \frac{y'_H}{y_H} \quad (8)$$

The free parameters c_1 and c_2 that appear in Eq. 2 can be expressed in function of the effective density parameters by:

$$\frac{c_1}{c_2} \approx 6 \frac{\tilde{\Omega}_x}{\tilde{\Omega}_m} \quad (9)$$

$$\frac{c_1}{c_2^2} = -\frac{f_{R_0}}{n} \left(\frac{12}{\tilde{\Omega}_m} - 9 \right)^{n+1} \quad (10)$$

These relations show that the free parameters of the model are $\tilde{\Omega}_m$, n , and f_{R_0} . The latter is constrained to $|f_{R_0}| \lesssim 0.1$ by solar system tests [5] and we will not investigate larger values in the next sections.

Solving the differential equations system for different values of n , f_{R_0} and $\tilde{\Omega}_m$, it is possible to obtain various evolution trends for the equation of state parameter w_x . In Fig.1 and Fig.2 we plot the behavior of w_x in function of

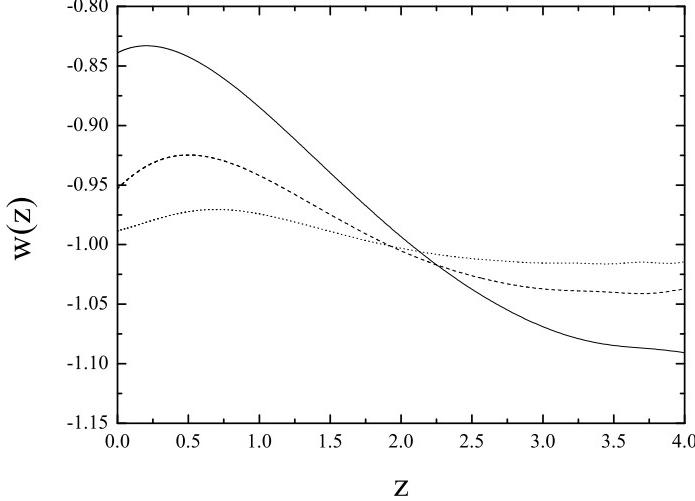


FIG. 1: The equation of state parameter w_x for $n = 1$ and f_{R_0} equal to -0.1 (solid line), -0.03 (dashed line) e -0.01 (dotted line). $\tilde{\Omega}_m$ is set to 0.24

the redshift z for different values of f_{R_0} and n .

As we can see in Fig.1 and Fig.2 the equation of state parameter w_x follows a peculiar behavior in function of the redshift. At the present time ($z = 0$) w_x has always a value higher than the one predicted by the Λ CDM model ($w = -1$) and, moving towards higher redshifts, it decreases crossing into the phantom region, i.e., assuming values lower than -1 . For even higher redshifts, w_x moves asymptotically towards -1 .

The same behavior is shown for any value of n and f_{R_0} , moreover decreasing the absolute value of f_{R_0} brings w_x closer to -1 , while increasing n shifts the phantom crossing at lower redshift.

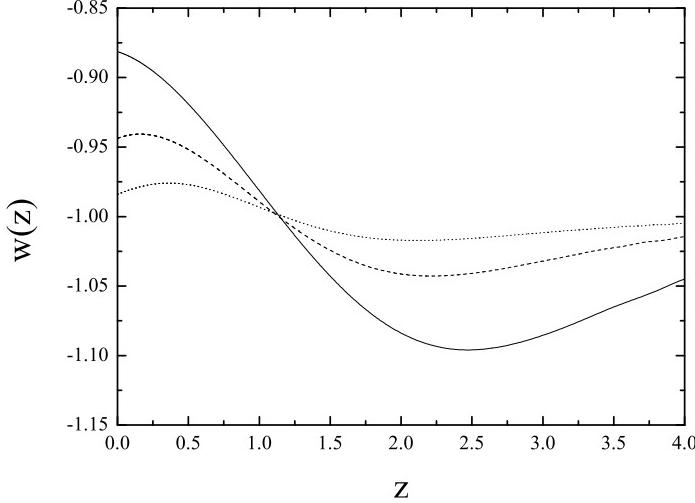


FIG. 2: The equation of state parameter w_x for $n = 2$ and f_{R_0} equal to -0.1 (solid line), -0.03 (dashed line) e -0.01 (dotted line). $\tilde{\Omega}_m$ is set to 0.24

The modification of the Einstein-Hilbert lagrangian brings to a new equation for the expansion of the Universe.

The predicted expansion must be consistent with standard cosmological results, i.e., should produce an accelerated era after radiation and matter dominance. Modified gravity models consistent with current observations, for example, should not change the scale factor evolution during the matter era. Hence, it is possible to derive general conditions for the cosmological viability of $f(R)$ theories. Introducing the parameters

$$m(r) = \frac{Rf_{RR}}{1+f_R} \quad r = -\frac{R(1+f_R)}{R+f} \quad (11)$$

it is possible to show [6] that for $f(R)$ theories the following conditions apply:

- The model has a standard matter era with a following accelerated phase if

$$m(r) \approx 0 \text{ and } m'(r) > -1 \quad \text{with } r \approx -1 \quad (12)$$

- The accelerated phase goes asymptotically towards the one produced by a dark energy with equation of state parameter $w = -1$, if

$$0 \leq m(r) \leq 1 \quad \text{for } r = -2 \quad (13)$$

- The expansion is not of the phantom type ($w < -1$) if

$$m(r) = -1 - r \quad (14)$$

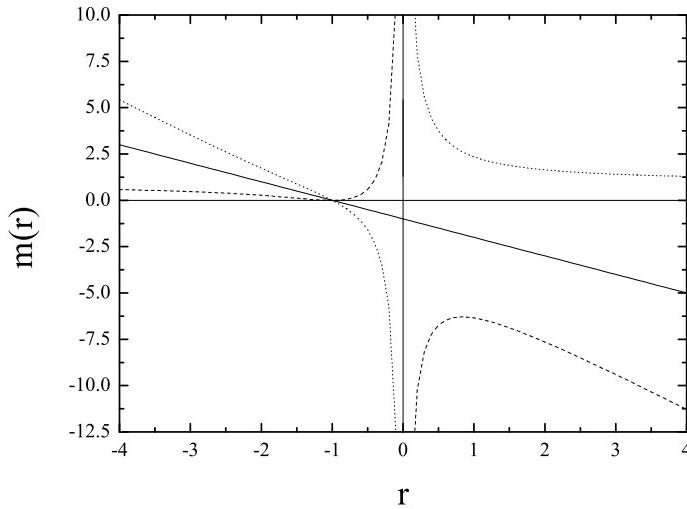


FIG. 3: Plot of the two solution for $m(r)$ obtained setting $n = 1$, $\tilde{\Omega}_m = 0.3$ and $\tilde{\Omega}_\Lambda = 0.7$. The solid line corresponds to the viable solution while the dashed line lies outside the viability region. The red line shows the solution $m(r) = -r - 1$.

It is possible to calculate $m(r)$ for the Hu and Sawicki model and to show the cosmological viability of this model. In Fig.3 we show that, for example, setting $n = 1$, $\tilde{\Omega}_m = 0.3$ and $\tilde{\Omega}_\Lambda = 0.7$, one obtains two solutions for $m(r)$, one living outside the viability region and the other corresponding to an acceptable expansion.

III. CONSTRAINTS ON THE HS MODEL

A. Method

In order to constrain the free parameters of the Hu and Sawicki model ($\tilde{\Omega}_m$, n and f_{R_0}), we predicted the expected theoretical values for a set of observables.

As now common in the literature, we considered the luminosity distance, defined by:

$$d_L(a) = \frac{1}{a} \int_a^1 \frac{da}{a^2 H(a)} = \frac{1}{aH_0} \int_a^1 \frac{da}{a^2 \sqrt{\tilde{\Omega}_m(y_H + a^{-3})}} \quad (15)$$

and the Hubble parameter:

$$H(a) = \sqrt{\tilde{\Omega}_m H_0^2 (y_H + a^{-3})} \quad (16)$$

Moreover, we also considered the quantity:

$$A \equiv \frac{\sqrt{\tilde{\Omega}_m}}{z_*} \left[z_* \frac{\Gamma^2(z_*)}{\epsilon(z_*)} \right]^{\frac{1}{3}} \quad (17)$$

where $z_* = 0.35$, $\Gamma(z_*) = \int_0^{z_*} dz/\epsilon(z)$ and $\epsilon(z) = H(z)/H_0$. The value of this parameter can be obtained from observations of Baryon Acoustic Oscillations (BAO) [7]. Hence, we have another way to compare model prediction with data.

We used the superovae data from Kowalski et al. [8] to obtain the observational trend of $d_L(z)$ and we considered $H(z)$ values obtained by Simon, Verde and Jimenez [9] and a prior on the Hubble parameter $H_0 = 0.72 \pm 0.08$ derived from measurements from the Hubble Space Telescope (HST, [10]). Finally, we used the value of A from Eisenstein et al. [11].

We compute a χ^2 variable for each observational quantity and then combine the results in a single variable $\chi^2 = \chi_{SN}^2 + \chi_{BAO}^2 + \chi_H^2 + \chi_{HST}^2$. Once the theoretical evolution of the observational quantities is defined, we can define a likelihood function as a function of n and f_{R_0} as

$$L = e^{-\frac{\chi^2 - \chi_{min}^2}{2}} \quad (18)$$

where χ_{min}^2 is the minimum value in the considered range of n and f_{R_0} .

B. Results

Combining the results obtained from the comparison between the experimental data for $H(z)$, A and $d_L(z)$ and their theoretical values, we can constrain the free parameters n and f_{R_0} for different values of $\tilde{\Omega}_m$ and $\tilde{\Omega}_x$.

Setting $\tilde{\Omega}_m = 0.2$ and $\tilde{\Omega}_x = 0.8$ it is possible to find an upper limit on n and on f_{R_0} , $n < 1.6$ and $f_{R_0} < -0.03$ at 2σ , while, performing the same analysis with different values of $\tilde{\Omega}_m$ and $\tilde{\Omega}_\Lambda$, we can see that raising $\tilde{\Omega}_m$ brings to more loosely constrained parameters.

We can note anyway that for higher values of $\tilde{\Omega}_m$, higher n are preferred, while smaller values of n are more in agreement with data for smaller $\tilde{\Omega}_m$.

As we can see from Figure 4, for $\tilde{\Omega}_m = 0.3$ both parameters are almost totally unconstrained; this points out the need of an independent measurement of the effective matter content ($\tilde{\Omega}_m$) in order to obtain some constraints on n and f_{R_0} .

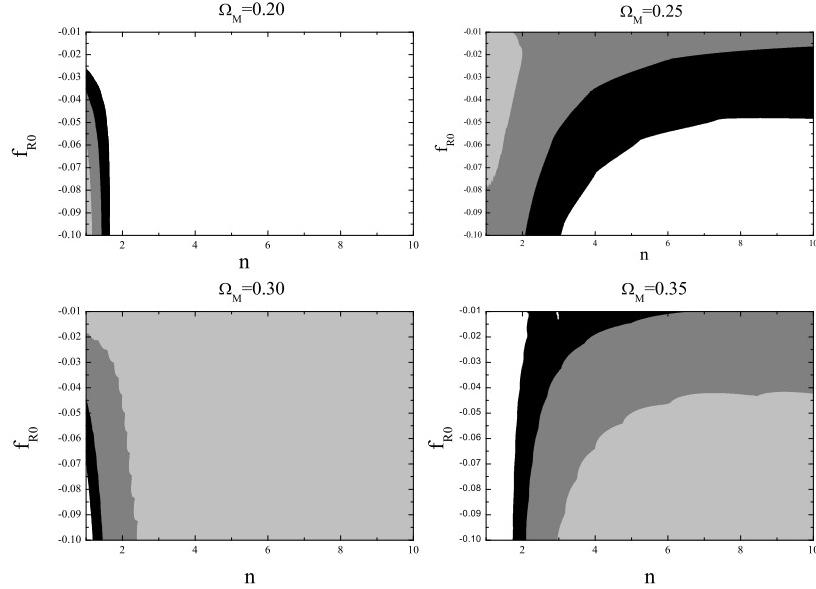


FIG. 4: 68%, 95% and 99% confidence levels in the $n-f_{R_0}$ plane in function of different values of $\tilde{\Omega}_m$.

It is interesting to compare the best fit values of χ^2 obtained in the modified gravity framework with the χ^2 of a cosmological constant model. In order to quantify the goodness-of-fit of the two models we use the Akaike information criterion (AIC) [12] and the Bayesian information criterion (BIC) [13], defined as

$$AIC = -2 \ln L + 2k$$

$$BIC = -2 \ln L + k \ln N$$

where L is the maximum likelihood, k the number of parameters and N the number of points.

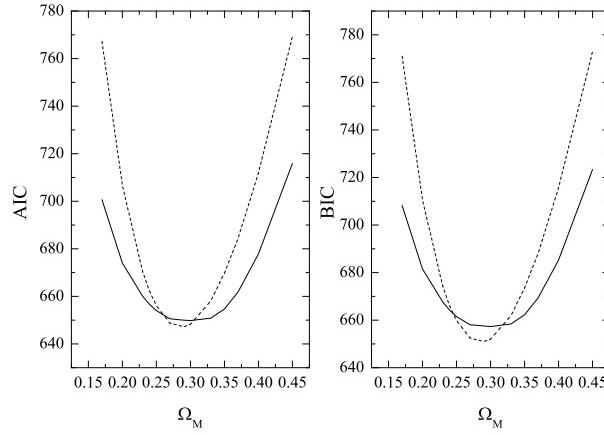


FIG. 5: AIC (left panel) and BIC (right panel) tests in function of Ω_m for the standard case of a cosmological constant (dashed line) and for the HS model (solid line).

In Fig. 5 we plot the best fit values of the AIC and BIC tests in function of $\tilde{\Omega}_m$ for the standard model based on a cosmological constant and the HS model respectively. As we can see, while the cosmological constant gives slightly better values for the overall best fit, when larger or smaller values of $\tilde{\Omega}_m$ are considered the AIC and BIC

tests provide definitely better values for the HS model. In few words, there is a weaker dependence of the observables considered from $\tilde{\Omega}_m$ in the case of HS scenario.

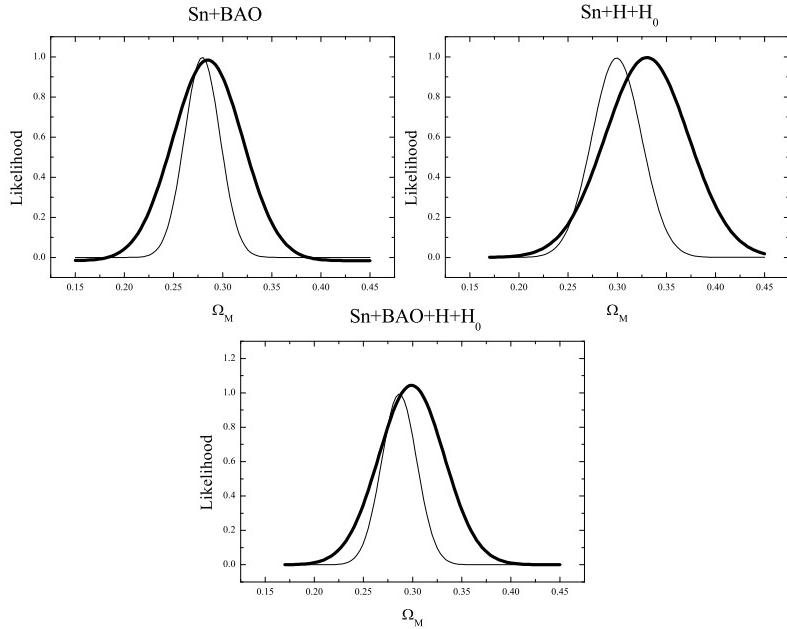


FIG. 6: The likelihood function $L(\tilde{\Omega}_m)$ for the Hu and Sawicki model (thicker lines) and ΛCDM model (fainter lines).

It is therefore important to quantify the impact of a different choice of the theoretical background model on the derived constraints on $\tilde{\Omega}_m$. In Fig. 6 we compare the constraints on the $\tilde{\Omega}_m$ parameter derived under the assumption of the HS scenario with the similar constraints but assuming general relativity and dark energy. As we can see the $\tilde{\Omega}_m$ parameter is less constrained respect to the ΛCDM scenario. This is certainly due to the larger amount of parameters present in the HS model. In the future, with the increasing experimental accuracy, if a discrepancy between independent constraints on the matter density will be found then a modified gravity scenario could be suggested as possible explanation. This result anyway shows that one should also be extremely careful in considering the current cosmological constraints because of their model dependence.

IV. CONCLUSIONS

In this paper we have compared a modified gravity scenario, the HS model, with several current cosmological datasets. We have found that the model is in excellent agreement with recent SN-Ia, BAO and $H(z)$ data. Moreover, the parameters of the model are substantially unconstrained by the data considered. This has important effect on the current constraints on some parameters as the matter density. We have shown that the assumption of the HS model enlarges the current constraints on this parameter by $\sim 30\%$. If a discrepancy between two experimental determinations of the matter density will be found in the framework of general relativity, then a possible solution could be the introduction of a modified gravity scenario. It will be duty of future experiments to scrutinize this interesting possibility.

Acknowledgments

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